Polygonizing Skeletal Sheets of CT-Scanned Objects by Partition of Unity Approximations

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ABSTRACT
The skeletal structures of solid objects play an important role in medical and industrial applications. Given a volumetrically sampled solid object, our method extracts a nice-looking skeletal structure represented as a polygon mesh. The purpose is to achieve a noise-robust extraction of the skeletal mesh from a real-world object obtained using a scanning technology such as the CT scan method. We first approximate the input through a set of spherically supported polynomials that provide an adaptively smoothed intensity field, and then perform a polygonization process to find the extremal sheet of the field, which is regarded as a skeletal sheet in this research. In our polygonization, a subset of the weighted Delaunay tetrahedrization defined by a set of spherical supports is used as an adaptively sampled grid. The derivatives for detecting extremality are analytically evaluated at the tetrahedron vertices.

1 INTRODUCTION
Skeletal-sheets have various applications in industrial applications. Recent advances in X-ray-based CT scanning technology have enabled accurate measurement of mechanical objects consisting of thin parts, as shown in the images on the left of Fig. 1. It is often necessary to extract the scanned object as single sheets (rather than as a two-sided thin object) for further digital manipulation such as reverse engineering, quality evaluation or physical simulation. Skeletal-sheet extraction is one possible solution to this problem. The goal of this research is to extract a polygon mesh approximating the skeletal sheets of a CT-scanned thin object as demonstrated in Fig. 1.

As reviewed in [1], many good methods for extracting skeletal structures have been developed so far. Given a voxelized solid, one method performs a thinning algorithm to obtain voxelized 1D/2D skeletons. These techniques extract skeletal structures as voxels, thus requiring a postprocessing if users desire an output as a mesh model. A polygonal approximation of the medial axis (i.e. skeletal sheets) can be directly obtained using the Voronoi diagram of sampling points on the object boundary. Unfortunately the topology of the resulting mesh is sensitive to noise on the boundary, thus this method is not suited for scanned objects, which typically have noisy boundaries. Using a potential field is a possible approach to be mainly used for extracting 1D skeletal curves. The skeletal points are extracted as the stationary points of a gradient descent flow.

Our skeletal mesh extraction is simply to find the maxima of a 3D scalar field defined by a function \( f(x) \) that is defined by a sum of local approximation functions inside a given solid. The scanning noise is effectively smoothed by error-driven support-size decision for local approximations without loss of important geometric features. The polygonization of skeletal sheets uses an adaptively sized tetrahedral grid obtained from the spherical supports of local approximations.

2 ALGORITHM
We take a set of spatial points equipped with normalized values \( \mathcal{P} = \{ p_i = (x_i, y_i) \mid x_i \in \mathbb{R}^3, y_i \in [0, 1] \} \) as input, such as CT scan data and grayscale images. This set consists of two kinds of point, i.e., those belonging to the object and those to the background. We assume that the points in the set \( \mathcal{P}_{obj} = \{ p_i \mid y_i > T \} \) represent the object and we refer to them as object points. Taking \( T \) as the user’s specification is reasonable because each material has a unique CT value.

First, our algorithm constructs a set of locally supported quadratic functions that approximate the values of \( p \in \mathcal{P} \).

Next, a tetrahedral mesh representing the object is generated by connecting the centers of the function supports.

Finally, the algorithm extracts the skeletal mesh. Points on the skeletal sheets are detected on the tetrahedral mesh edges. A small patch is generated around a detected skeletal point. The set of patches becomes the skeletal mesh.

Approximation. We generate a \( C^2 \)-continuous approximation function following [3]. The function is represented as a partition of unity based on polynomial approximations, i.e.,

\[
f(x) = \sum \phi_i(x)g_i(x),
\]

where \( x = (x, y, z) \in \mathbb{R}^3 \) and \( g_i \) is a second-order equation. Expressing the radius and the center of the support of \( g_i \) in \( r_i \) and \( c_i \), the weight function \( \phi(x) \) is described as

\[
\phi(x) = \frac{w_r(||x - c||)}{\sum_j w_r(||x - c_j||)},
\]

\[
w_r(d) = \begin{cases} \exp(-d^2/2r^2)/(2\pi)^{3/2}r^2 & (d \leq r_i) \\ 0 & (\text{otherwise}) \end{cases}
\]
The generation of $\{g_i\}$, a set of local approximations in $f$, is summarized as below.

1. Set $\mathcal{P}_{ob}$ as a list of candidate support centers denoted by $\mathcal{C}$.
2. Select a point $p_i$ randomly from $\mathcal{C}$ and decide the support radius and function.
3. Remove points in the support from $\mathcal{C}$. If $\mathcal{C} = \emptyset$ then the algorithm terminates, otherwise go back to step 2.

It must be noted that the support radius is adapted to the distribution of values in the vicinity of the center and the number of generated functions is much lower than the number of input points.

To generate a covering of $\mathcal{P}_{obj}$, we remove some points inside the convex hull defined by the object points in this support. In 2D, a similar method is proposed in [4], and we extend it to 3D.

Tetrahedrization. After the generation of the supports, connecting their centers, we obtain a tetrahedrization to be a subset of weighted Delaunay tetrahedrization.

In this triangulation, some skinny elements (with a low aspect ratio) appear around the boundaries of the object. This is because some small spheres are consumed by larger spheres, meaning that their centers cannot appear as vertices. The mesh quality can be improved by smoothing the size changes of elements. Multiplying all radii of supports by $\alpha < 1$ and using these new radius values solving this problem and more vertices appear in the generated mesh. Good results are obtained using $\alpha = 0.75$ in all examples in this paper. Such scaling enables a better quality of tetrahedrization.

Skeletal mesh extraction. First, we find a point on each edge, that is in the maxima of the scalar field. In this step, in order to extract a smooth skeletal mesh, we use the gradient $\mathbf{g} = \nabla f$ and Hessian $H = \nabla^2 \mathbf{g}$ of the approximation function. They are estimated well with the help of the set of the local approximation functions. The maximal point satisfies

$$\langle e, g \rangle = 0, \quad \lambda < 0$$

where $e$ is the eigenvector of $H$ associated with the minimum eigenvalue $\lambda$.

Let $p_1, p_2$ be the end points of an edge and $x_1, x_2$ the coordinates. See the image on the left of Fig. 2. Let the eigenvectors corresponding to the minimum eigenvalues of Hessian $H$ at the vertices $p_1$ and $p_2$ be $e_1$ and $e_2$. We assume the inner product $\langle e_1, e_2 \rangle > 0$. If this is not true, flip $e_2$ to make $-e_2$ and to satisfy this condition. The existence of zero-crossing of $\langle e, g \rangle$ is tested by the condition

$$\langle e_1, g \rangle \langle e_2, g \rangle < 0.$$

Next, for edges that satisfy the above condition, we test whether the extremum is the maximum or minimum. We use a condition proposed in [2] for this test: if the condition

$$\langle e_i, g \rangle \langle (x - x_i), e_j \rangle > 0$$

holds for $(i, j) = (1, 2)$ or $(2, 1)$, the edge has a skeletal point $c$. This condition means if a maxima exists between these endpoints, the values of $f$ increase toward it along $e$ (see the left image of Fig. 2). By assuming that $\langle e, g \rangle$ changes linearly along the edge, the coordinate of $c$ is calculated as the internally dividing point.

Next, a skeletal patch is made around $c$ as shown in the image on the right of Fig. 2. A patch consists of at most $k$ triangles sharing $c$. $k$ is the number of tetrahedra incident to the edge on which $c$ is found. A vertex of a patch, $q_i$, is the centroid of skeletal points on the edges of the $i$-th tetrahedron.

To obtain well-connected and not-fragmented skeletal patches, smoothing the fields of derivatives is an effective choice. We can easily obtained such a smoothing effect by enlarging the support sizes as $\{ \sigma r_i \}$ ($\sigma > 1$) before evaluating $\mathbf{g}$ and $H$, which means the local approximations are spread out more large regions. The all results demonstrated in this paper are obtained by $\sigma = 4$.

3 RESULTS AND DISCUSSION

Some results for CT scan data are shown in Figs. 1 and 3. Each of extracted skeletal meshes reaches to the end of a thin object without branching. In Fig. 3, the bumps on the surface of the original object do not affect the result sheet.

4 CONCLUSION AND FUTURE WORK

In this paper, we have proposed an algorithm to extract the skeletal meshes of CT-scanned thin objects using a set of approximation functions and an adaptive grid. This algorithm can generate a smooth surface mesh of the skeletal sheet directly from raw data and the sheet extends almost to the end of the object.

The improvement of crossing-point detection is a left issue. Theoretically speaking, our algorithm can handle even T-junctions. But actually we observed that the derivatives around such parts are very unstable, hence detection for non-manifold parts is still a difficult issue.

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REFERENCES


