Polygonizing Skeletal Sheets of CT-Scanned Objects by Partition of Unity Approximations

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Overview
The skeletal structures of solid objects play an important role in medical and industrial applications.
Given a volumetrically sampled solid object, our method extracts a nice-looking skeletal structure represented as a polygonal mesh. Our purpose is to achieve a noise-robust extraction of the skeletal meshes from real-world objects obtained using a scanning technology such as CT scanners.
We first approximate the input through a set of spherically supported polynomials that provide an adaptively smoothed intensity field, and then apply a polygonization process to find the extremal sheet of the field, which is regarded as a skeletal sheet in this research.

Examples

1. Approximation
Approximate the CT scanned value by a \( C^2 \)-continuous function
\[
f(x) = \sum \phi_i(x) g_i(x) : \mathbb{R}^3 \to \mathbb{R}
\]
through the partition of unity approach. Iteratively generate a local approximation quadric \( g_i(x) \) with a spherical support, whose center is \( c_i \) and the radius is \( r_i \), until the object is covered by the set of the supports. \( r_i \) is adapted to the distribution of CT scanned values in the vicinity of \( c_i \).

We use a following weighting function
\[
\phi_i(x) = \frac{w_i(x)}{\sum w_j(x)}
\]
\[
w_i(x) = \frac{\exp \left( -\|x - c_i\|^2 / 2r_i^2 \right)}{\sqrt{2\pi}r_i} \begin{cases} 1 & \|x - c_i\| \leq r_i \\ 0 & \text{otherwise} \end{cases}
\]

2. Tetrahedrization
Generate a subset of weighted Delaunay tetrahedrization. The generators are the centers and the weights are quadric of the radii of supports. This mesh is used as an adaptively sampled grid.

3. Skeletal mesh extraction
Find the maximal point on each edge of the tetrahedral mesh by the conditions:
\[
\begin{cases}
(e_1, g_1) \langle e_2, g_2 \rangle < 0 \\
(e_1, g_1) \langle e_1, (x_j - x_i) \rangle > 0 \\
\text{for } (i, j) = (1, 2) \text{ or } (2, 1)
\end{cases}
\]
The derivatives are analytically evaluated at the tetrahedron vertices.
Finally generate triangles sharing the maximal point to be a polygonized skeletal structure.